Calculus Solutions to Section 3.1

1) $f'(2) = \lim_{h \to 0} \dfrac{\frac{1}{2+h} - \frac{1}{2}}{h} = \lim_{h \to 0} \dfrac{\frac{2-(2+h)}{2(2+h)}}{h} = \lim_{h \to 0} \dfrac{\frac{-h}{2(2+h)}}{h} = \lim_{h \to 0} \dfrac{-1}{2(2+h)} = -\dfrac{1}{4}$

2) $f'(1) = \lim_{h \to 0} \dfrac{(1+h)^2 + 4 - 5}{h} = \lim_{h \to 0} \dfrac{1 + 2h + h^2 - 1}{h} = \lim_{h \to 0} \dfrac{h(2+h)}{h} = \lim_{h \to 0}(2+h) = 2$

3) $f'(-1) = \lim_{h \to 0} \dfrac{3-(-1+h)^2 - 2}{h} = \lim_{h \to 0} \dfrac{3-(1-2h+h^2)-2}{h} = \lim_{h \to 0} \dfrac{2h - h^2}{h} = \lim_{h \to 0}(2-h) = 2$

4) $f'(0) = \lim_{h \to 0} \dfrac{(0+h)^3 + (0+h) - 0}{h} = \lim_{h \to 0} \dfrac{h^3 + h}{h} = \lim_{h \to 0} \dfrac{h(h^2+1)}{h} = \lim_{h \to 0}(h^2+1) = 1$

5) $f'(2) = \lim_{x \to 2} \dfrac{\frac{1}{x} - \frac{1}{2}}{x-2} = \lim_{x \to 2} \dfrac{\frac{2-x}{2x}}{x-2} = \lim_{x \to 2} \dfrac{-1}{2x} = -\dfrac{1}{4}$

6) $f'(1) = \lim_{x \to 1} \dfrac{x^2 + 4 - 5}{x-1} = \lim_{x \to 1} \dfrac{x^2 - 1}{x-1} = \lim_{x \to 1}(x+1) = 2$

7) $f'(3) = \lim_{x \to 3} \dfrac{\sqrt{x+1} - 2}{x-3} = \lim_{x \to 3} \dfrac{\sqrt{x+1} - 2}{x-3} \bullet \dfrac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2} = \lim_{x \to 3} \dfrac{x+1-4}{(x-3)\left(\sqrt{x+1}+2\right)} = \lim_{x \to 3} \dfrac{1}{\left(\sqrt{x+1}+2\right)} = \dfrac{1}{4}$

8) Do you really need to use a formula for this?

9) Or this one? 10) Or this one?

11) $\dfrac{d}{dx}\left(x^2\right) = \lim_{h \to 0} \dfrac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \dfrac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \dfrac{h(2x+h)}{h} = \lim_{h \to 0}(2x+h) = 2x$

12) $\dfrac{d}{dx}\left(3x^2\right) = \lim_{h \to 0} \dfrac{3(x+h)^2 - 3x^2}{h} = \lim_{h \to 0} \dfrac{3x^2 + 6xh + 3h^2 - 3x^2}{h} = \lim_{h \to 0} \dfrac{h(6x+3h)}{h} = \lim_{h \to 0}(6x+3h) = 6x$

13) b 14) a 15) d 16) c

17) Tangent: $y - 3 = 5(x - 2)$; Normal: $y - 3 = -\dfrac{1}{5}(x-2)$

18)

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{2(x+h)^2 - 13(x+h) + 5 - (2x^2 - 13x + 5)}{h} = \lim_{h \to 0} \frac{2\left(x^2 + 2xh + h^2\right) - 13x - 13h + 5 - 2x^2 + 13x - 5}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2 - 13h}{h} = \lim_{h \to 0}(4x + 2h - 13) = 4x - 13$$

The slope at $x = 3$ is -1: Equation of the tangent line is $y - 16 = -1(x - 3)$

19) a) Slope of tangent line at (1,1)

$\lim_{x\to 1}\frac{x^3-1}{x-1} = \lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{x-1} = \lim_{x\to 1}(x^2+x+1) = 3$, so, an equation of the tangent line is $y - 1 = 3(x - 1)$.

b) Equation of the normal at (1,1) is $y-1=-\frac{1}{3}(x-1)$.

20) a) Slope of tangent line at (4,2)

$\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4} = \lim_{x\to 4}\frac{\sqrt{x}-2}{(\sqrt{x}-2)(\sqrt{x}+2)} = \lim_{x\to 4}\left(\frac{1}{\sqrt{x}+2}\right) = \frac{1}{4}$, so, an equation of the tangent line is $y-2=\frac{1}{4}(x-4)$.

b) Equation of the normal at (4,2) is $y-2=-4(x-4)$.